

## CHOICE OF DIAGNOSTIC DECISION MAKING IN MEDICINE AND INTERVENTION MISTAKE PREDICTION USING MATHEMATICAL MODELS

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**Abstract:** *Most processes, found in medicine, are nonlinear, chaotic, have a high level of complexity. The decisions in health care are often stereotyped, managed by habits preferences, previous experience and official directives. These decisions might be not completely conscious. There are a lot of papers, devoted to modeling diagnostics or treatment conduction, but still behavior responses of medical practitioners were not studied, no universal comprehensive and effective model was created. Besides nonlinear nature of biomedical phenomena, pathologies, its chaotic expression, all the information process in medicine at each of its stages, including information perception by available diagnostic tools, analysis, decision making and implementation of therapeutic interventions, are complex, chaotic. We made attempts to integrate this process, bringing scheme into harmony. Each stage requires creation some mathematical model, that might be described by generalized equation. These equations can be substituted into one, that could be solved in closed system. We do not aim to find some absolute kind of decision, its statistically calculated optimal way of solution, but accent on a special mood, the state of expert, which could give a possibility to make only one correct decision with failure in input parameters. In such cases the lack of prior data is compensated by doctor's experience.*

**Keywords:** imaging, mathematical modeling, intervention, choice, error analysis, Monty Hall paradox, method of branches and boundaries.

### **ACM Classification Keywords:**

H. Information Systems: H.1 MODELS AND PRINCIPLES: H.1.0 General;

G.1.0 Mathematics of Computing General *Error analysis*;

G.2 DISCRETE MATHEMATICS: G.2.1 Combinatorics: *Combinatorial algorithms*

G.2.2 Graph Theory.

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### **Introduction**

The decisions in health care are often stereotyped, managed by habits preferences, previous experience and official directives. Creating a reliable mathematical models and use of information technology at all stages of the treatment process from the expression the pathological processes to implementation of therapeutic interventions associated with neurophysiological perception of these phenomena, making decisions in the absence of input parameters for creation self-controlled systems based on forecasts of future medical errors are important tasks [1]. Although mathematical models can not replace human judgment in the field of medicine, they can be useful and crucial to make only one correct decision in the absence of information on input parameters. Until now its often compensated by doctor's experience.

**A. Logistic model selection diagnostic decisions in medicine.** In the clinical setting, often there are situations, when there is a need for a solution that lies in the choice between several equally probable options.

This medical decisions, based on data from scientific studies that have presented statistical calculations, e.g., accuracy, specificity, predicative importance of diagnostic methods and effectiveness of treatment. For lack of input parameters previous experience of the doctor is used and an intuitive decision is made. Often, after the selection process for diagnosis or treatment there is additional information that does not directly affect the pre-selection process. Such information is often ignored by doctors, is not used to correct medical decisions. To ensure and optimize logically and intellectually controlled diagnostic process we propose scheme for appropriate choice of diagnostic decisions.

We suggest the use of logistic models of Monty Hall paradox and its generalization (in the case of four factors) [2, 3] for optimizing diagnostic decisions in medicine [4]. As certainly, Monty Hall paradox (for three factors) in the primary (classical) formulation taught by the example of three doors. We formulate the statement in more usual language. There are three boxes. In one of the boxes are expensive valuables, and in the other two - less. Two people take part in the performance: the person who chooses the casket (participant), and the person conducting the procedure of choice (presenter). Participant selects one of three boxes in the first step. After that, the presenter chooses among the two remaining boxes, which have smaller values and opens it. He offers to change participant's choice and select new box that has not yet been elected. The question arises: *do not increase the probability to choose casket of precious values, if the participant choosing the proposal will lead?* Thus, **YES** it will increase and it will be as high as **2/3**. If the participant's selection will not be changed, the probability that the casket, which he chose is expensive, has value of **1/3** [2]. This finding contradicts the everyday intuitive perception of most people, so this problem is called *Monty Hall paradox*.

Monty Hall paradox itself is applied to the case of three boxes. There is a practical need to consider the case of four boxes. There are four boxes. In one of the boxes are expensive valuables, and in the other three - smaller values. For this task logistic model selection boxes offer the best values in two stages.

**The first stage of the model.** Participant selection in the first step chooses one of the four boxes. Three boxes remain that have not yet elected. The presenter chooses (among the three / not yet chosen) box with a smaller value and opens it. Participant is proposed to change the choice and selects the small box with two that have not yet chosen. If he does not change his original choice, the procedure of choice ends. He gets casket, which he chose from the beginning. If he agrees with the change of the initial choice, then - *go to the second stage model*.

**The second stage of the model.** Presenter proposes to make the final choice of two boxes that were not elected at the first stage and choose one of them opening it. In this logistic model for the four boxes is an element of paradox. If he does not change his original choice this time, the probability that the casket, which he chose from the beginning, containing winning value is **0.25**. If he changed the initial choice and the second phase selects one of two boxes remaining, the probability of win will be equal to **0.375** [3].

**Example (treatment algorithm) [4].** Condition: the treatment started conducting according to one scheme, should not be changed. Several (3 or 4) equivalent circuits according to input parameters are possible. Additional information (such as laboratory tests), which is not directly related to the selected scheme, does not indicate the correct circuit, but can eliminate one or two circuits, while not affecting the choice between those that remained. Change according to the previous selection leads to increase the probability of correct choice from **1/3 to 2/3** (for 3 equivalent schemes), or from **0.25 to 0.375** (equivalent to 4 circuits), for these conditions simulated situation. The features inherent to modern medicine indicate that the appearance of new additional excluding parameters, based on ignorance of the obviously negative option, is often the most randomized. Ignorance of correct choice (additional information regarding all options simultaneously), for example, increases the probability not to **2/3**, but only to **1/2** for the three schemes.

**2. The study of adverse negative prognostic parameters of interventional mistakes using the scheme of the method of branches and boundaries.** Conducting minimally invasive interventions under radiology / ultrasound control requires continuous improvement of multidisciplinary approach to the analysis of errors and develop a differentiated approach to each clinical situation for achieve the efficiency about 100%. Previously we reported [5,6] to solve combinatorial (correctable) problem of selection options of negative prognostic indicators for interventional radiology / sonography mistakes to ensure a high level of patient safety, as well as study-level skills and minimal training required for training programs for interventional medicine (in particular in pain management) by applying the method of branches and boundaries. From the formal (mathematical) point of view the problem of negative options selection of prognostic indicators for interventional sonography mistakes is a discrete-combinatorial. Finding "good" solutions for such problems usually are resistive in nature. In mathematical terms the problem of finding solutions to these problems are called in the theory of optimal solutions of discrete optimization problems.

**Experimental studies.** According to the goal we included 2 groups of physicians: 6 anesthesiologists, who had no previous experience in interventional sonography and a group of experts (ultrasound doctors) - 6 people with previous experience in interventions under ultrasound control. Fundamental difference between skills level of ultrasound doctors were excluded, all studies were conducted in relative isolation. Ultrasound examination was carried out using a portable ultrasound device Sonosite M-Turbo with multifrequency linear and convex probes (used in hospital operating room). The study was conducted on special designed phantoms, which included a gel phantom, phantom and biological electronic device to record accurate needle penetration into the object. All professionals - ultrasound doctors (experts) and anesthesiologists (novices) performed 30 punctures of each group of studies. The comparative study of different methods of introducing the needle to different kinds of phantoms was conducted, recording performance, mistakes were determined, statistical analysis was performed. In case of absence of experimental data for the formation of separate branches of the graph the expert method according to clinical experience of two independent experts was applied.

One approach to solving discrete optimization problems are algorithmic scheme of the method of branches and boundaries. For the first time this method was proposed by Land and Doig [7] in 1960 for solving integer linear programming. When applying algorithmic scheme of the method of branches and boundaries to solve a specific class of discrete optimization to use mathematical characteristics and specificity of this class of problems that often allows us to develop efficient numerical algorithms for the special method of branches and boundaries to address these problems. At the core algorithmic scheme of the method of branches and boundaries is the idea of successive breaking the current set of admissible solutions to a subset (a subset of branching). At each step of this method of partitioning elements (ie subsets of solutions) are checking to determine whether this subset contain the optimal solution. Verification carried out by calculating the value of the lower estimates (lower bounds) objective function (for minimization problem) or the upper estimate (estimates down) objective function (for maximization problem) in this subset of solutions and comparing the value assessment of the value of record at the moment. *Record* - is currently the best objective function value of the found solutions.

For the problem of maximizing algorithmic scheme of the method of branches and boundaries will be as follows. If the upper bound of objective function for this subset of solutions is more (less) record, this subset may be rejected for further consideration, since it obviously does not contain an optimal solution (it is not "promising" for further consideration. Record value will change, if the objective function for the new solution found less than previously estimated record, this new found. If at some step can discard all the elements of partition (a subset of solutions), the record value - an optimal solution of the initial value problem . Otherwise, with subsets of solutions

that are not rejected, was elected one of the "promising" and it is divided into subsets of branching. These new solutions again tested a subset of "optimality" and so on, until at some step does not work, meaning that a record will be higher (not lower) values of upper bounds of objective function on all subsets of branching. the end of the computation process and record the current value is the optimal value of objective function and the corresponding solution is the optimal solution of original problem.

The method of branches and boundaries includes two components of treatment: the construction of branches and computation limits (upper) values of the function objective optimization. Branching - is to identify all possible options so as not to leave without loss of any option. We're building a tree, all branches (branching).When you start branching in any situation, the detected branches contain all possible ways of development of the situation. The main requirement is that these subsets do not overlap and their union would have created a whole set of options for solutions. If not cut off branches to complete their analysis, the method branches stood to be exhaustive of all options.

The second component of the procedure of the method of branches and boundaries - the definition and use of boundaries (top) values of the function purpose - to assess their branches without detailed analysis and cut-off "unpromising." Must be, at any time of analysis, numerical rating desired objective function value.

In general, the discrete optimization problem is formulated as follows: to find optimum (maximum of) functions, where the element is selected from some discrete set, is considered such a problem:

$$F(x) \rightarrow \min, \quad (1)$$

$$x \in X, \quad (2)$$

where  $X$  - is a discrete set.

The study problem (the problem of the choice options of negative prognostic indicators mistakes interventional sonography) is regarded as a discrete optimization problem (in fact, a problem on a graph) and its solution using algorithmic scheme of the method of branches and boundaries, and procedures strings of finding sequences of branches (by arcs) on the graph [5.6].

The task of selecting variants of negative prognostic indicators of error is interventional sonography optimization (maximization) as a functional solution - the probability of its realization. This problem has a discrete nature and relates to the so-called - full of problems. Find the solution of such problems is resistive in nature and is very time-consuming computing process.

The function aims will be to maximize the likelihood of a decision (the way from the beginning of the branching tree for sequences of branches to the top of the latest branches), that is.

$$\text{MAX } F((i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)) = PR(p_{12}x p_{23}x \dots x p_{k-1, k}) \quad (3)$$

Where  $(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)$  - a sequence of arcs of the tree-graph partial variants (single) decisions are the way to the top Initial (tree roots) to the top of the latest in this way (since it does not have arcs that would come out) - the corresponding probabilities of (appearance) arc path and - the operator multiplying the relevant numbers.

To construct the relevant pathways that are solving the problem (3) are used as mathematical and information technology finding the shortest admissible paths in the graph [8].

**Results.**

All doctors, who participated in the study, were succeeded in the imaging and intervention trials. The results of the punctures and registered errors of interventions are presented in Fig. 1 (corresponding to part of the graph).

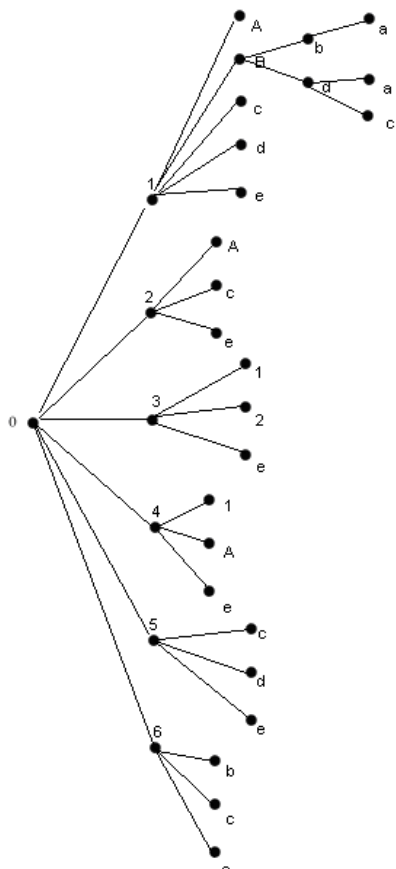


Fig. 1. A. Tree (graph) of errors while performing interventions – regional anesthesia under ultrasound guidance (picture is presented reduced without excessive branching options).

Thus, on a tree (graph) is shown the algorithm of options (string) negative consequences when performing regional anesthesia with admitted intervention and imaging errors, where

- 1 - loss of visualization a needle;
- 2 - loss of visualization an object;
- 3 - incorrect mapping of testing area and images on the screen;
- 4 - poor selection of a needling place;
- 5 - uneven distribution of local anesthetic;
- 6 - fatigue;
- A - incorrect needle trajectory
- B - damage to surrounding tissue
- a - no effect;
- b - reduced quality of anesthesia;

- c - longer duration of manipulation;
- d - alarm continued correction (eg, third injection).
- e - fast adequate correction;

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## Conclusion

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Using these logistic models in clinical practice should optimize the clinical algorithms processing to reduce stereotypical judgments and redundant diagnostic and therapeutic medical procedures. The modeled mistakes led to a decrease in the quality of intervention, but could cause iatrogenic injury in clinical conditions. The method of branches and boundaries effectively solves the problem of choice and interventional mistakes negative prognostic indicators as a discrete optimization problem.

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